

文章编号: 1000-8055(2004) 04-0755-07

镍基单晶合金弹塑性本构模型

智平^{1,2}, 刘义伦², 尹泽勇³, 杨治国³, 成晓鸣³

(1. 株洲工学院 机械工程学院, 湖南 株洲 412008;

2. 中南大学 机电工程学院, 湖南 长沙 410083;

3. 中国航空动力机械研究所, 湖南 株洲 412002)

摘要: Hill 模型用于面心立方单晶的弹塑性变形, 在描述屈服特性的晶向相关性时精度不高。考虑正交各向异性材料在偏轴加载时存在拉、剪应力耦合效应的影响, 通过增加一项由应力偏张量分量的二次项乘积构成的应力不变量, 对 Hill 屈服模型进行修正, 并根据单晶合金的屈服特点提出了新的单晶合金屈服准则。用该屈服准则对 DD3 单晶合金的屈服应力进行预测, 760℃ 时能很好地符合试验结果; 与 Hill 屈服准则比较, 预测精度显著提高。在此基础上, 重新定义了适合新屈服准则的等效应力, 并由联合流动法则, 以屈服函数作为塑性势函数, 采用等向硬化模型, 建立了单晶合金的弹塑性本构模型, 推导出相应的弹塑性矩阵。对于各向同性材料, 新屈服准则及其等效应力则退化为 Von Mises 屈服准则和其相应的等效应力。

关键词: 航空、航天推进系统; 单晶合金; 正交各向异性; 屈服准则; 弹塑性本构模型

中图分类号: V 231. 91; O 344. 2

文献标识码: A

Study of Elastic-Plastic Constitutive Model for Single Crystal Nickel-Based Superalloy

DING Zhi-ping^{1,2}, LIU Yi-lun², YIN Ze-yong³,

YAN Zhi-guo³, CHENG Xiao-ming³

(1. School of Mechanical and Engineering,

Zhuzhou Institute of Technology, Zhuzhou 412008, China;

2. School of Mechanical and Electrical Engineering,

Center South University, Changsha 410083, China;

3. China Aviation Power Plant Research Institute, Zhuzhou 412002, China)

Abstract: The correlation of crystal orientation to yield characteristic of FCC single crystal is not good when Hill's (1948) yield criterion is used to describe the elastic-plastic deformation of FCC single crystal. Taking into account the effects that the components of tension stress are coupled with components of torsion stress when off-axis loads are applied to orthotropic material, Hill's yield criterion for plastically orthotropic solids is modified by adding an invariant which is composed of quadratic product item of components of deviatoric stress tensor, as a results, a new yield criterion for single crystal Nickel-based superalloy is put forward in the present paper in terms of the characteristics of single crystal Nickel-based superalloy. Using the new criterion to

收稿日期: 2003- 12- 01; 修订日期: 2004- 04- 27

基金项目: 湖南省自然科学基金资助项目(02JJY 5014)

作者简介: 丁智平(1956-), 男, 湖南桃江县人, 中南大学机电工程学院博士生, 研究员, 主要从事机械结构强度、机械设计和优化设计研究。

predict the yield stresses of single crystal Nickel-based superalloy DD3, the correlation to experiments is very good and more accurate than that of Hill's at 760°C. Equivalent stress adapted to the new criterion is defined. Thinking of the yield function as a plastic potential function from the associated flow rule and with reference to the isotropic hardening model, the elastic-plastic constitutive model for single crystal Nickel-based superalloy is constructed and the corresponding elastic-plastic matrix is educed. New yield criterion and its equivalent stress will be reduced to Von Mises yield criterion and corresponding equivalent stress for isotropic material.

Key words: aerospace propulsion system; single crystal superalloy; orthotropy; yield criterion; elastic-plastic constitutive model

镍基单晶高温合金因其优越的高温抗疲劳、抗蠕变性能,成为航空发动机热端部件的重要材料。研究单晶合金的高温力学性能,其中本构模型的研究是一个重点。由于单晶合金力学行为的各向异性,增加了单晶合金本构关系研究的难度。已经开展的单晶合金本构模型研究概括起来可以分为两大类^[1~8]:其一是基于材料细观塑性理论的晶体滑移本构模型研究,其二是基于宏观实验现象的唯象连续本构模型研究。前者从滑移系的激活和滑移规律出发,前提是必须准确掌握滑移规律。由于滑移规律本身非常复杂,对外界条件敏感,所建立的模型中包含许多与滑移系相关的材料常数,需做大量复杂的试验,特别是至今尚缺乏一种好的方法来确定反映滑移系之间相互作用的潜硬化系数,因此在工程实际应用中受到一定的限制。而唯象连续本构模型因其计算比较简单,在工程实际中用于真实构件的结构强度和寿命的初步计算时,仍受到欢迎。在进行正交各向异性材料的弹塑性分析时,由 Hill R 于 1948 年提出的 Hill 屈服模型^[9],因其简单实用,受到较为广泛的应用,但其不足之处是模型没有考虑正交各向异性材料在偏轴加载时存在拉、剪应力耦合,因而在描述单晶材料屈服特性的晶向相关性时精度不高。本文提出了一个考虑拉、剪应力耦合能量分量的单晶合金屈服准则,经过试验验证具有较高的精度。在此基础上,建立单晶合金的弹塑性本构方程,并推导出相应的弹塑性矩阵。

1 单晶合金的弹性本构关系

对于正交各向异性材料,由线弹性连续介质的广义虎克定律可得:

$$\{\epsilon\} = [S]\{\sigma\} \tag{1}$$

式(1)中:

$$\{\sigma\} = [\sigma_{11} \ \sigma_{22} \ \sigma_{33} \ \sigma_{23} \ \sigma_{31} \ \sigma_{12}]^T$$

$\{\epsilon\} = [\epsilon_{11} \ \epsilon_{22} \ \epsilon_{33} \ \gamma_{23} \ \gamma_{31} \ \gamma_{12}]^T$
[S]为6×6的柔度矩阵。镍基单晶材料是正交各向异性材料的一个特例,3个晶轴方向[001],[010],[100]具有同等的力学行为。取材料主轴为坐标轴,则:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{44} \end{bmatrix} \tag{2}$$

式(2)中, $S_{11}=1/E$, $S_{44}=1/G$, $S_{12}=-\mu/E$ 。将式(1)用张量表示,写成增量形式:

$$d\epsilon_{ij} = S_{ijkl} d\sigma_{kl} \tag{3}$$

2 单晶合金的塑性本构关系

2.1 屈服准则和等效应力

文献[10]提出了一种单晶合金的屈服准则,它通过增加一项应力不变量来考虑单晶合金偏轴加载时存在的拉、剪应力耦合效应:

$$\Omega = \sigma^2 = A[(\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2] + B(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) + C(\sigma_{23}^2\sigma_{11} + \sigma_{31}^2\sigma_{22} + \sigma_{12}^2\sigma_{33}) \tag{4}$$

从式(4)中的第3项可以看出,这项应力不变量仅包含了3个剪应力分量与对应的3个正应力分量的耦合效应。实际上每一个剪应力分量均与3个正应力分量产生耦合效应,因此,上式在考虑拉、剪应力耦合效应时还不够充分。由塑性力学原理可知,应力偏张量与材料的塑性变形密切相关,因此可以设想单晶合金的屈服函数是由若干应力偏张量的不变量的线性组合构成。根据单晶合金的屈服特性,假设:(1)忽略静水压力对材料屈服应力的影响(对金属材料而言,实验证明静水压力

对材料屈服影响可以不考虑^[11]); (2) 忽略包辛格效应, 材料拉伸和压缩性能相同(单晶合金具有立方对称结构, 其主要变形机制是滑移, 它沿正、反两个方向的作用近似相等^[12]); (3) 单晶合金沿材料主轴[100], [010]和[001]方向的性能相同。由以上假设, 材料的屈服函数可表示为:

$$\begin{aligned}\Omega = & \sigma_s^2 = a(S_x^2 + S_y^2 + S_z^2) + \\ & b(S_{yz}^2 + S_{zx}^2 + S_{xy}^2) + \\ & c(S_{yz}^2 S_x^2 + S_{zx}^2 S_y^2 + S_{xy}^2 S_z^2)\end{aligned}\tag{5}$$

式中的 a, b 和 c 为与温度相关的材料各向异性参数, S_i 为应力偏张量, 等式右边的各项均为应力偏张量的不变量, 第 3 项用来考虑材料在偏轴受载时产生的拉、剪应力耦合作用。将应力偏张量用应力分量表示, 并取 $A = a/6, B = b, C = c/9$ 则式(5)变为:

$$\begin{aligned}\Omega = & \sigma_s^2 = A[(\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + \\ & (\sigma_{11} - \sigma_{22})^2] + B(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) + \\ & C[\sigma_{23}^2(2\sigma_{11} - \sigma_{22} - \sigma_{33})^2 + \\ & \sigma_{31}^2(2\sigma_{22} - \sigma_{11} - \sigma_{33})^2 + \\ & \sigma_{12}^2(2\sigma_{33} - \sigma_{11} - \sigma_{22})^2]\end{aligned}\tag{6}$$

式中, σ_i 为材料主轴方向的正应力; σ_{ij} 为材料主轴方向的剪应力。比较式(4)与式(6), 可知后者右边的第 3 项中包含了每一个剪应力分量与其它 3 个正应力分量的耦合效应, 充分考虑了单晶合金偏轴受载时存在的拉、剪应力耦合效应。若令 $C = 0$, 则(6)式退化为 Hill 屈服准则。对各向同性材料, $S_{[001]} = S_{[111]} = S_{[011]} = \sigma_s$, 可得 $A = 1/2, B = 3, C =$

0。将这些数值代入式(6)得:

$$\begin{aligned}\Omega = & \sigma_s^2 = \frac{1}{2}[(\sigma_{22} - \sigma_{33})^2 + \\ & (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2] + \\ & 3(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\end{aligned}\tag{7}$$

这就是 Von Mises 屈服准则。为了确定在复杂应力状态下单晶合金是否进入屈服, 为进一步进行构件的弹塑性应力分析提供依据, 定义由式(6)确定屈服函数的单晶合金的等效应力为:

$$\begin{aligned}\bar{\sigma} = & \{A[(\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + \\ & (\sigma_{11} - \sigma_{22})^2] + B(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) + \\ & C[\sigma_{23}^2(2\sigma_{11} - \sigma_{22} - \sigma_{33})^2 + \\ & \sigma_{31}^2(2\sigma_{22} - \sigma_{11} - \sigma_{33})^2 + \\ & \sigma_{12}^2(2\sigma_{33} - \sigma_{11} - \sigma_{22})^2]\}^{1/2}\end{aligned}\tag{8}$$

用最小二乘法拟合式(8)中的各向异性参数, 对 DD3 单晶合金的屈服应力进行预测, 计算结果与材料数据^[13]对比见表 1。

用 Hill 屈服准则预测 DD3 单晶合金的屈服应力, 计算结果见表 2^[10]。

由表 1 和表 2 的结果看出, 用本文提出的单晶合金屈服准则计算 DD3 单晶合金的屈服应力, 在[001]方向上与试验值完全吻合, 在[111], [112]和[011]3 个方向上的误差很小, 温度为 760℃时[011]和[112]方向的屈服应力, 与 Hill 模型相比, 误差分别降低 8.98%和 6.41%, 预测精度显著提高。

表 1 新屈服准则预测 DD3 单晶合金屈服应力

Table 1 New yield criterion predict the yield stress of single crystal DD3

温度/℃	950			850			760		
A	0.5			0.5			0.5		
B	2.930 5			2.655 9			3.584 4		
$C \times 10^{-4}$	- 1.298			0.008 6			- 1.094 4		
	试验值	计算值	误差/ %	试验值	计算值	误差/ %	试验值	计算值	误差/ %
$S_{[001]}$ /MPa	525	525	0	870	870	0	915	915	0
$S_{[011]}$ /MPa	490	496.17	1.26	840	832.48	- 0.89	899	876.23	- 2.53
$S_{[111]}$ /MPa	510	518.89	1.74	830	818.58	- 1.37	1 030	1 000.2	- 2.89
$S_{[112]}$ /MPa	530	512.47	- 3.31	809	831.98	2.84	880	946.38	7.54

表 2 Hill 屈服准则预测 DD3 单晶合金的屈服应力

Table 2 Hill's yield criterion predict the yield stress of single crystal DD3

温度/ °C	950			850			760		
	试验值	计算值	误差/ %	试验值	计算值	误差/ %	试验值	计算值	误差/ %
$S_{[001]}/\text{MPa}$	525	525	0	870	870	0	915	915	0
$S_{[111]}/\text{MPa}$	510	510	0	830	830	0	1 030	1 030	0
$S_{[011]}/\text{MPa}$	490	513. 79	4. 85	840	840. 18	0. 021	899	1002. 5	11. 51
$S_{[112]}/\text{MPa}$	530	513. 79	- 3. 1	809	840. 18	3. 84	880	1002. 5	13. 95

2. 2 塑性本构关系

根据Druker 公设,塑性应变增量可表示为:

$$d\epsilon_{ij}^p = d\lambda \frac{\partial \varphi}{\partial \sigma_{ij}}$$

当 $\varphi = 0$ 且 $\frac{\partial \varphi}{\partial \sigma_{ij}} d\sigma_{ij} = 0$ 时 $d\lambda \geq 0$

当 $\varphi < 0$ 或 $\varphi = 0$

但 $\frac{\partial \varphi}{\partial \sigma_{ij}} d\sigma_{ij} < 0$ 时 $d\lambda = 0$

(9)

其中, φ 为塑性势。在 Drucker 公设成立的前提下, 由联合流动法则, 取塑性势等于屈服函数, 有:

$$\varphi = \bar{\sigma} - \sigma_s = 0$$

式中 σ_s 为材料的屈服极限。上式代入式(9), 得:

$$d\epsilon_{ij}^p = 2d\lambda \bar{\sigma} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} \tag{10}$$

式中, $d\lambda > 0$ 为非负比例系数。为了确定系数 $d\lambda$, 需要引入关于材料应变硬化的假设。对于硬化材料, 当其进一步产生塑性变形时, 需要消耗一定的塑性功, 用 dw_p 表示塑性功增量, 则有:

$$dw_p = \sigma_{ij} d\epsilon_{ij}^p$$

将式(10)代入上式, 得:

$$\frac{\partial \bar{\sigma}}{\partial \sigma_{11}} = \frac{1}{\bar{\sigma}} A (2\sigma_{11} - \sigma_{22} - \sigma_{33}) +$$

$$C [2\sigma_{23}^2 (2\sigma_{11} - \sigma_{22} - \sigma_{33}) - \sigma_{31}^2 (2\sigma_{22} - \sigma_{33} - \sigma_{11}) - \sigma_{12}^2 (2\sigma_{33} - \sigma_{11} - \sigma_{22})]$$

$$\frac{\partial \bar{\sigma}}{\partial \sigma_{22}} = \frac{1}{\bar{\sigma}} A (2\sigma_{22} - \sigma_{33} - \sigma_{11}) +$$

$$C [2\sigma_{31}^2 (2\sigma_{22} - \sigma_{33} - \sigma_{11}) - \sigma_{12}^2 (2\sigma_{33} - \sigma_{11} - \sigma_{22}) - \sigma_{23}^2 (2\sigma_{11} - \sigma_{22} - \sigma_{33})]$$

$$\frac{\partial \bar{\sigma}}{\partial \sigma_{33}} = \frac{1}{\bar{\sigma}} A (2\sigma_{33} - \sigma_{11} - \sigma_{22}) +$$

$$C [2\sigma_{12}^2 (2\sigma_{33} - \sigma_{11} - \sigma_{22}) - \sigma_{23}^2 (2\sigma_{11} - \sigma_{22} - \sigma_{33}) - \sigma_{31}^2 (2\sigma_{22} - \sigma_{33} - \sigma_{11})]$$

$$\frac{\partial \bar{\sigma}}{\partial \sigma_{33}} = \frac{1}{\bar{\sigma}} [B\sigma_{23}^2 + C\sigma_{23}^2 (2\sigma_{11} - \sigma_{22} - \sigma_{33})^2]$$

$$dw_p = 2d\lambda \sigma_{ij} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} \bar{\sigma} \tag{11}$$

($i, j = x, y, z$)

同时, 若用 $d\bar{\epsilon}$ 表示等效塑性应变增量, 那么还存在:

$$dw_p = \bar{\sigma} d\bar{\epsilon} \tag{12}$$

比较式(11)和式(12), 可得:

$$2d\lambda = \frac{1}{\sigma_{ij} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}}} d\bar{\epsilon} \tag{13}$$

($i, j = x, y, z$)

由材料拉伸和压缩性能相同的假设, 采用等向硬化模型, 硬化条件为:

$$\bar{\sigma} = H \left[\int d\bar{\epsilon} \right]$$

微分上式得:

$$d\bar{\sigma} = H d\bar{\epsilon} \quad \text{或} \quad d\bar{\epsilon} = \frac{d\bar{\sigma}}{H}, \tag{14}$$

其中, H 为等向硬化模量。将式(14)代入式(15)

得:

$$2d\lambda = \frac{d\bar{\sigma}}{\sigma_{ij} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} H},$$

($i, j = x, y, z$) (15)

由式(8)可得 $\partial \bar{\sigma} / \partial \sigma_{ij}$ 的各分量为:

$$\frac{\partial \bar{\sigma}}{\partial \sigma_1} = \frac{1}{\bar{\sigma}} [B\sigma_{31} + C\sigma_{31}(2\sigma_{22} - \sigma_{33} - \sigma_{11})^2]$$

$$\frac{\partial \bar{\sigma}}{\partial \sigma_2} = \frac{1}{\bar{\sigma}} [B\sigma_{12} + C\sigma_{12}(2\sigma_{33} - \sigma_{11} - \sigma_{22})^2] \quad (16)$$

$$\sigma_{ij} \frac{\partial \bar{\sigma}}{\partial \sigma_j} = \sigma_{11} \frac{\partial \bar{\sigma}}{\partial \sigma_1} + \sigma_{22} \frac{\partial \bar{\sigma}}{\partial \sigma_2} + \sigma_{33} \frac{\partial \bar{\sigma}}{\partial \sigma_3} + \sigma_{23} \frac{\partial \bar{\sigma}}{\partial \sigma_{23}} + \sigma_{31} \frac{\partial \bar{\sigma}}{\partial \sigma_{31}} + \sigma_{12} \frac{\partial \bar{\sigma}}{\partial \sigma_{12}} = \bar{\sigma}$$

$$(i, j = x, y, z) \quad (17)$$

将式(17)代入式(15), 得:

$$2d\lambda = \frac{d\bar{\sigma}}{H \bar{\sigma}}$$

将上式代入式(10), 并考虑式(14)后, 可得:

$$d\epsilon_{ij}^p = \frac{1}{H} \left(\frac{\partial \bar{\sigma}}{\partial \sigma_j} d\bar{\sigma} \right) \quad (18a)$$

$$d\epsilon_{ij}^p = \frac{\partial \bar{\sigma}}{\partial \sigma_j} d\bar{\epsilon}_p \quad (18b)$$

其中 $\partial \bar{\sigma} / \partial \sigma_j$ 由式(16)确定。式(18)就是基于式(8)导出的单晶材料的塑性本构方程。

3 单晶合金的弹塑性本构关系

因为 $d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p$, 将式(3)和式(18)合并得到单晶材料的弹塑性本构方程:

$$\left. \begin{aligned} d\epsilon_{ij} &= S_{ijkl} d\sigma_{kl} + \frac{1}{H} \left(\frac{\partial \bar{\sigma}}{\partial \sigma_j} d\bar{\sigma} \right) \\ d\epsilon_{ij} &= S_{ijkl} d\sigma_{kl} + \frac{\partial \bar{\sigma}}{\partial \sigma_j} d\bar{\epsilon}_p \end{aligned} \right\} \quad (19)$$

其中 $[C] = [S]^{-1}$ 为弹性矩阵。在(23)式两边同时左乘 $\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T$, 得:

$$\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T d\{\sigma\} = \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] (d\{\epsilon\} - d\{\epsilon_p\}) \quad (24)$$

因为:

$$d\bar{\sigma} = \frac{\partial \bar{\sigma}}{\partial \sigma_{11}} d\sigma_{11} + \frac{\partial \bar{\sigma}}{\partial \sigma_{22}} d\sigma_{22} + \frac{\partial \bar{\sigma}}{\partial \sigma_{33}} d\sigma_{33} + \frac{\partial \bar{\sigma}}{\partial \sigma_{23}} d\sigma_{23} + \frac{\partial \bar{\sigma}}{\partial \sigma_{31}} d\sigma_{31} + \frac{\partial \bar{\sigma}}{\partial \sigma_{12}} d\sigma_{12} = \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T d\{\sigma\}$$

利用式(14), 有:

$$\text{则式(24)可化为: } H \left(d\bar{\epsilon}_p \right) = \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] d\{\epsilon\} - \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} d\bar{\epsilon}_p$$

由此即得等效塑性应变增量与全应变增量的关系式:

$$d\bar{\epsilon}_p = \frac{\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C]}{H \left(1 + \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \right)} d\{\epsilon\} \quad (25)$$

将式(25)代入到式(18b)得:

$$d\{\epsilon\}_p = \frac{\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C]}{H \left(1 + \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \right)} d\{\epsilon\} \quad (26)$$

4 单晶合金的弹塑性矩阵

设:

$$\left. \begin{aligned} d\{\sigma\} &= \begin{bmatrix} d\sigma_{11} & d\sigma_{22} & d\sigma_{33} & d\sigma_{23} & d\sigma_{31} & d\sigma_{12} \end{bmatrix}^T \\ d\{\epsilon\} &= \begin{bmatrix} d\epsilon_{11} & d\epsilon_{22} & d\epsilon_{33} & d\epsilon_{23} & d\epsilon_{31} & d\epsilon_{12} \end{bmatrix}^T \end{aligned} \right\} \quad (20)$$

分别为应力增量与应变增量向量。材料进入塑性后, 应力—应变关系可用矩阵方程表示为:

$$d\{\sigma\} = [C]_p d\{\epsilon\} \quad (21)$$

式中, $[C]_p$ 为弹塑性矩阵。下面来具体推导 $[C]_p$ 。

全应变增量可表示为弹性应变增量与塑性应变增量之和:

$$d\{\epsilon\} = d\{\epsilon_e\} + d\{\epsilon_p\} \quad (22)$$

因此:

$$d\{\sigma\} = [C] (d\{\epsilon\} - d\{\epsilon_p\}) \quad (23)$$

将式(26)代入到式(23)得:

$$d\{\sigma\} = \left[[C] - \frac{[C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C]}{H' + \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}} \right] d\{\epsilon\} \quad (27)$$

比较式(27)与式(21), 可得弹塑性矩阵为:

$$[C]_p = [C] - \frac{[C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C]}{H' + \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}} \quad (28)$$

下面给出 $[C]_p$ 的显式表达式。由式(8)可得:

$$\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T = \frac{1}{\bar{\sigma}} [\sigma'_{11} \quad \sigma'_{22} \quad \sigma'_{33} \quad \sigma'_{23} \quad \sigma'_{31} \quad \sigma'_{12}]$$

其中:

$$\left. \begin{aligned} \sigma'_{11} &= A(2\sigma_{11} - \sigma_{22} - \sigma_{33}) + \\ &C[2\sigma_{23}^2(2\sigma_{11} - \sigma_{22} - \sigma_{33}) - \sigma_{31}^2(2\sigma_{22} - \sigma_{33} - \sigma_{11}) - \sigma_{12}^2(2\sigma_{33} - \sigma_{11} - \sigma_{22})] \\ \sigma'_{22} &= A(2\sigma_{22} - \sigma_{33} - \sigma_{11}) + \\ &C[2\sigma_{31}^2(2\sigma_{22} - \sigma_{33} - \sigma_{11}) - \sigma_{12}^2(2\sigma_{33} - \sigma_{11} - \sigma_{22}) - \sigma_{23}^2(2\sigma_{11} - \sigma_{22} - \sigma_{33})] \\ \sigma'_{33} &= A(2\sigma_{33} - \sigma_{11} - \sigma_{22}) + \\ &C[2\sigma_{12}^2(2\sigma_{33} - \sigma_{11} - \sigma_{22}) - \sigma_{23}^2(2\sigma_{11} - \sigma_{22} - \sigma_{33}) - \sigma_{31}^2(2\sigma_{22} - \sigma_{33} - \sigma_{11})] \\ \sigma'_{23} &= B\sigma_{23} + C\sigma_{23}(2\sigma_{11} - \sigma_{22} - \sigma_{33})^2 \\ \sigma'_{31} &= B\sigma_{31} + C\sigma_{31}(2\sigma_{22} - \sigma_{33} - \sigma_{11})^2 \\ \sigma'_{12} &= B\sigma_{12} + C\sigma_{23}(2\sigma_{33} - \sigma_{11} - \sigma_{22})^2 \end{aligned} \right\}$$

而:

$$\left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] = \frac{1}{\bar{\sigma}} [S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_6]$$

式中:

$$S_1 = \frac{E}{(1+\mu)(1-2\mu)} [(1-\mu)\sigma'_{11} + \mu\sigma'_{22} + \mu\sigma'_{33}]$$

$$S_2 = \frac{E}{(1+\mu)(1-2\mu)} [\mu\sigma'_{11} + (1-\mu)\sigma'_{22} + \mu\sigma'_{33}]$$

$$S_3 = \frac{E}{(1+\mu)(1-2\mu)} [\mu\sigma'_{11} + \mu\sigma'_{22} + (1-\mu)\sigma'_{33}]$$

$$S_4 = G\sigma'_{23}, \quad S_5 = G\sigma'_{31}, \quad S_6 = G\sigma'_{12}$$

$$[C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] = \frac{1}{\bar{\sigma}^2} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} [S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_6]$$

$$\text{令: } A = \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} = \frac{1}{\bar{\sigma}^2} (S_1\sigma'_{11} + S_2\sigma'_{22} + S_3\sigma'_{33} + S_4\sigma'_{23} + S_5\sigma'_{31} + S_6\sigma'_{12})$$

塑性矩阵可表示为:

$$[C]_p = \frac{[C] \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\} \left\{ \frac{\partial \bar{\sigma}}{\partial \sigma} \right\}^T [C]}{H' + A} = \frac{1}{\bar{\sigma}^2 (H' + A)} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} [S_1 \ S_2 \ S_3 \ S_4 \ S_5 \ S_6] \quad (29)$$

则弹塑性矩阵为:

$$[C]_q = [C] - [C]_p \quad (30)$$

5 结 论

(1) 由应力偏张量分量的二次项乘积构成的应力不变量考虑正交各向异性材料在偏轴受载时存在的拉、剪应力耦合效应, 对 Hill 屈服准则进行修正, 根据单晶合金的屈服特点, 提出工程上实用的新单晶合金屈服准则。

(2) 新屈服准则中的各向异性参数可通过 3 个不同取向的单向拉伸试验确定。用该屈服准则对 DD3 单晶合金的屈服应力进行预测, 760℃ 时能很好地符合试验结果; 与 Hill 屈服准则比较, 预测精度显著提高, 能够更好地描述单晶合金屈服特性的取向相关性。

(3) 根据新屈服准则, 重新定义了等效应力。若不考虑拉、剪应力耦合的能量分量, 新屈服准则及其等效应力的退化形式与 Hill 屈服准则用于单晶合金的形式相同; 对于各向同性材料, 新屈服准则及其等效应力退化为 von Mises 屈服准则和相应的等效应力。

(4) 由联合流动法则, 采用等向硬化模型, 建立了单晶合金的弹塑性本构模型, 并导出相应的弹塑性矩阵。

参考文献:

[1] Swanson G A, Linask I, Nissley D M, et al. Life Prediction and Constitutive Models for Engine Hot Section Anisotropic Materials[R]. NASA - CR- 179594, 1987.

[2] Meric L, Poubanne P, Cailletaud G. Single Crystal Modeling for Structural Calculations: Part 1- Model Presentation[J]. *ASME Journal of Engineering Materials and Technology*, 1991, 113: 162- 170.

[3] YIN Ze-yong, CHENG Xiao-ming, YANG Zhi-guo, et al. Study on Strength and Life of Anisotropic Single Crystal Blade- Part 1: Crystallographic Constitutive Models and Applications[J]. *Chinese Journal of Aeronautics* 2001, 14: 18- 23.

[4] YUE Zhu-feng, LU Hen-house, ZHENG Chang-qing, et al. A Viscoplastic Crystallographic Analysis of A Nickel-Based Sin-

gle Crystal Blade[J]. *Journal of Propulsion Technology*, (In Chinese) 岳珠峰, 吕震宙, 郑长卿, 尹泽勇. Ni 基单晶叶片粘塑性分析. 推进技术, 1996, 17(3): 50- 55.

[5] YUE Zhu-feng, ZHENG Chang-qing, YIN Ze-yong, et al. The Study of the Creep Constitutive Equation and the Life Predictive Model of Single Crystal Nickel- based Superalloys [J]. *Journal of Mechanical Strength*, (In Chinese) 岳珠峰, 郑长卿, 尹泽勇, 杨治国. Ni 基单晶超合金的蠕变本构方程及寿命预测模型研究. 机械强度, 1994, 16(4): 2- 5.

[6] ZHOU Bai-zhuo, ZHANG Xiao-xia, LUO Yanming. A Unified Viscoplastic Constitutive Model of Orthogonal Anisotropic Material[J]. *Journal of Propulsion Technology*, (In Chinese) 周柏卓, 张晓霞, 罗焰明. 正交各向异性材料粘塑性统一本构模型. 推进技术, 1998, (1): 89- 93.

[7] Mitsuru Kanda, Masao Sakane, Masateru Ohnami. High Temperature Multiaxial Low Cycle Fatigue of CM SX- 2 Ni-Base Single Crystal Superalloy[J]. *ASME Journal of Engineering Materials and Technology*, 1997, 119: 153- 160.

[8] Mac Lachlan D W, Knowles D M. Fatigue Behaviour and Lifing of Two Single Crystal Superalloys[J]. *Fatigue Fract Engng Mater Struct*, 2001, 24: 503- 521.

[9] Hill R. The Mathematical Theory of Plasticity[M]. London: Oxford University Press, 1950.

[10] DING Zhi-ping, LIU Yi-lun, YIN Ze-yong, et al. Study of Yield Criterion for Single Crystal Nickel-Based Superalloys [J]. *Journal of Mechanical Strength*, 2004, 26(1) (In Chinese) 丁智平, 刘义伦, 尹泽勇, 杨治国, 成晓鸣. 镍基单晶高温合金的屈服准则研究. 机械强度, 2004, 26(2): 175- 179.

[11] HE Jun-yi, LIN Xiang-du. Numeric Solution for Nolinear Problem of Engineering Structure[M]. Peking: Publishing Company of National Defence Industry, 1994. (In Chinese) 何君毅, 林祥都. 工程结构非线性问题的数值解法. 北京: 国防工业出版社, 1994.

[12] Liu C, Huang Y, Stout M G. On the Asymmetric Yield Surface of Plastically Orthotropic Materials: A Phenomenological Study[J]. *Acta Mater*, 1997, 45(6): 2397- 2406

[13] Chinese Aviation Material Manual (Secondly Edition) Vol. 2, Edited by Committee to Edit Chinese Aviation Material Manual[M]. Peking: China standard Publishing Company, 2002. 5(In Chinese) 中国航空材料手册(第 2 版) 第二卷, 中国航空材料手册编辑委员会编, 北京: 中国标准出版社出版, 2002.